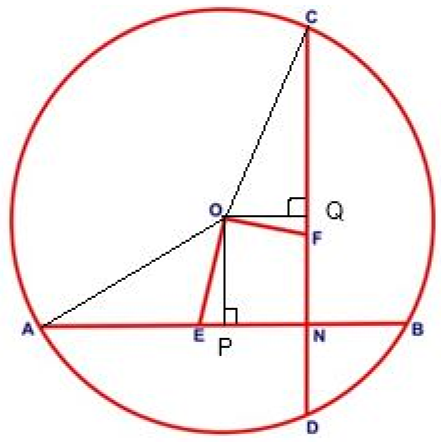

Given: $AB \perp CD$,

NB = NE

ND = NF

To Prove: $OE^2 - OF^2 = 2(NE^2 - NF^2)$

Construction: $OP \perp AB$, $OQ \perp CD$, and AO, OC joined.

Proof: As per construction $OP \perp AB$

$\Rightarrow$ OP bisect AB

$\Rightarrow AP = PB = PN + NB = NE - EP + NB$

$\Rightarrow AP = NE - EP + NE$ (as NE=NB given)

$\Rightarrow AP = 2NE - EP$ -----------------(1)

Again as per construction $OQ \perp CD$

$\Rightarrow$ OQ bisect CD

$\Rightarrow CQ = QD = QF + FN + ND$

$\Rightarrow CQ = QF + FN + FN$ (as FN=ND given)

$\Rightarrow CQ = 2NF + QF$ -----------------(2)

AO = OC (radius of circle)

$\Rightarrow AO^2 = OC^2$

$\Rightarrow AP^2 + OP^2 = CQ^2 + OQ^2$ (as per construction OAP and COQ are right triangle)

$\Rightarrow (2NE - PE)^2 + QN^2 = (2FN + QF)^2 + PN^2$ as per construction OPNQ is a rectangle.

(OP = QN, OQ = PN) & (1) , (2)

$\Rightarrow (2NE - PE)^2 + (QF + FN)^2 = (2FN + QF)^2 + (NE - PE)^2$

$\Rightarrow 4NE^2 + PE^2 - 4NE.PE + QF^2 + FN^2 + 2QF.FN = 4FN^2 + QF^2 + 4QF.FN + NE^2 + PE^2 - 2NE.EP$

$\Rightarrow 3NE^2 - 3NF^2 = 2QF.FN + 2NE.EP$ -----------------(3)

Now

$OE^2 - OF^2 = \left(OP^2 + PE^2\right) - \left(OQ^2 + QF^2\right)$

$\Rightarrow OE^2 - OF^2 = OP^2 + PE^2 - OQ^2 - QF^2$

$\Rightarrow OE^2 - OF^2 = NQ^2 + PE^2 - NP^2 - QF^2$ (OP = QN, OQ = PN)

$\Rightarrow OE^2 - OF^2 = NQ^2 - QF^2 + PE^2 - NP^2$

$\Rightarrow OE^2 - OF^2 = (NF + QF)^2 - QF^2 + PE^2 - (NE - EP)^2$

$\Rightarrow OE^2 - OF^2 = NF^2 + QF^2 + 2QF.NF - QF^2 + PE^2 - NE^2 - EP^2 + 2NE.EP$

$\Rightarrow OE^2 - OF^2 = NF^2 - NE^2 + 2QF.NF + 2NE.EP$

$\Rightarrow OE^2 - OF^2 = NF^2 - NE^2 + 3NE^2 - 3NF^2$ (by using 3)

$\Rightarrow OE^2 - OF^2 = 2(NE^2 - NF^2)$ Proved.